THE EFFECTS OF NOISE ON POLARIMETRIC SAR DATA

A. Freeman

Jet propulsion Laboratory
California Institute of Technology
4800 Oak Grove Drive
Pasadena, CA 91109, USA,
Tel: (818) 3541887
Fax: (818) 3936943
e-mail: Freeman@jplpol.jpl.nasa.gov

ABSTRACT

Polarimetric SAR data can provide a great deal of information about the scattering behavior of the surface under observation. Polarimetric SAR systems often measure the scattering matrices of the areas under observation in linear polarizations (H and V). From the scattering matrix commonly used forms such as the covariance matrix and the Stokes matrix can be easily derived. Other measures derived from polarimetric SAR data include correlation coefficients between scattering matrix terms and the mode and variance of phase differences between scattering matrix terms. The effects of additive system noise on these measurements is not often considered in the literature on this subject.

In this paper, the effects of additive system noise on measurements derived from polarimetric SAR data will be examined. It will be shown how first-order noise effects can be removed and how second-order noise effects can be reduced for some measurements. Some commonly occurring characteristics of polarimetric SAR data which may be attributed to noise, such as the. pedestal on a polarization signature, or a broadening in the distribution of the HH-VV phase difference over an area, or a reduction in the magnitude of a correlation coefficient, will be identified. The appearance of azimuth ambiguities in polarimetric SAR data is also addressed.

Keywords: Polarimetric SAR, Data Analysis, Azimuth ambiguities

BASIC POLARIMETRIC SAR MEASUREMENTS

For nominally calibrated polarimetric SAR measurements, the measured matrix, M, should be related to the scattering matrix of interest, S, via:

$$M = \begin{pmatrix} S_{HH} & S_{VH} \\ S_{HV} & S_{VV} \end{pmatrix} + \begin{pmatrix} n_{HH} & n_{VH} \\ n_{HV} & n_{VV} \end{pmatrix} \equiv S + N$$

where N is a matrix representing the noise in each polarization channel. This noise cannot be removed from the measurements. It remains to characterize this additive noise, examine the sources which give rise to it and its effects on measures commonly derived from polarimetric SAR data.

NOISE SOURCES

Potential sources of additive noise in polarimetric SAR data are:

- Thermal noise - from the radar receiver, antenna and background radiation (e.g. earth noise)
- Analog-to-Digital (ADC) conversion noise - includes both quantization (rounding or truncation) and saturation noise
- Interference - due to transponders on the ground or to transmitters
- Ambiguities - 'ghost' images visible in both range and azimuth directions

In this paper, saturation noise and interference will not bc included in the discussion.

SNR GOALS

Goals for Signal-to-Noise ratios in polarimetric SAR data are 20dB [1].

NOISE CHARACTERISTICS

Thermal noise and quantization noise are usually characterized as white noise inputs. "i'hat is, they have constant spectral density over all bandwidths. We characterize this type of noise term" as having two-dimensional (real and imaginary), zero-mean, Gaussian distributions, with the following properties:

$$\langle n_{jk} \rangle = 0$$
$$\langle n_{jk} n_{jk}^* \rangle = \sigma_{jk}^*$$
$$\langle n_{jk} n_{lm}^* \rangle = 0 \text{ , for } j \neq l \text{ or } k \neq m.$$
$$\langle n_{jk} S_{lm}^* \rangle = 0 \text{ , for any j, k, l, m.}$$

where σ_{jk}^n is the noise power (or noise-equivalent sigma-zero) in the polarization channel jk. We assume that the noise terms are uncorrelated with each other and with the scattering matrix (signal) terms.

The assumption of constant spectral density for thermal noise and quantization noise may not apply after SAR processing, in which several filters (azimuth matched filter, range matched filter, multi-looking, etc.) are applied. The net effect of these filters is usually to leave the noise spectrum shaped instead of constant over some bandwidth. This is known as colored noise.

AMBIGUITIES

Ambiguities are caused by aliasing in the azimuth dimension and by receiving echoes from different pulses simultaneously in the range dimension. Ambiguities are unlike other forms of 'noise' in that they can appear to be focused and look like 'ghost' images. Azimuth ambiguities occur at fixed along-track repeat positions with respect to the position, x_o, of the actual feature, i.e. at positions:

$$x = x_o + \frac{n \lambda R_o PRF}{V} \equiv x_o + n\Delta A$$

where n is an integer denoting the number of the ambiguity, λ is the wavelength, R_o the range at closest approach, V the relative speed between platform and target and PRF the pulse repetition frequency.

When collecting polarimetric SAK data, some systems, such as the NASA/JPL AIRSAR, collect HH, HV and Vii, VV returns separately, i.e. on adjacent pulses. The returns are separated by intervals 1/2PRF in time or V/2PRF in the along-track dimension, where 2PRF is the frequency at which pulses arc transmitted, but PRF is the frequency at which H or V polarized pulses are transmitted.. If' the HH, HV response occurs at position x_o, the VH, VV response occurs at position

$$x = x_o + \frac{V}{2PRF} = x_o + \Delta x$$

The Vii, W returns arc [hen resampled so that they arc registered with the HH, HV returns.

The azimuth phase history for a target positioned at x_o can be represented by:

$$s(x) = \exp\left\{j\frac{4\pi R_o}{\lambda}\right\}\exp\left\{j\frac{2\pi}{\lambda R_o}(x - x_o)^2\right\}$$

Suppose the HH, HV returns are from a target positioned at x_o. After resampling the VH, VV returns will still have a phase shift $2\pi(\Delta x)^2/\lambda R_o$ with respect to the HH, HV returns. It is straightforward to remove this phase shift with an appropriate multiplication by a complex number.

Now consider what happens to azimuth ambiguous returns when the above procedure is applied. The HH, HV ambiguous responses occur at the position $x_o + n\Delta A$, while the VH, VV ambiguous responses occur at $x_o + n\Delta A + \Delta x$. Using the expression for the azimuth phase history given in (3), it can be shown that, after resampling and correction for the nominal phase shift, the phase difference between the ambiguous HH, HV and the Vii, VV returns is $n\pi$. This can readily be seen in AIRSAR images of bright point targets: the first ambiguities lie at a distance AA either side of the main response and the phase difference between HH and W is π radians. Thus the first (and strongest) ambiguities often look like 'ghosts' of the real thing but with the HH-VV phase difference changed by --180 degrees. In Figure 1 is shown a point target HH response from a corner reflector at a calibration site, together with the 1st ambiguity on the right. The peak level of the ambiguity is 22dB down from the peak of the actual response. The HH-VV phase difference for the peak of the ambiguity is 168°, while the corresponding value for the actual response is -6°.

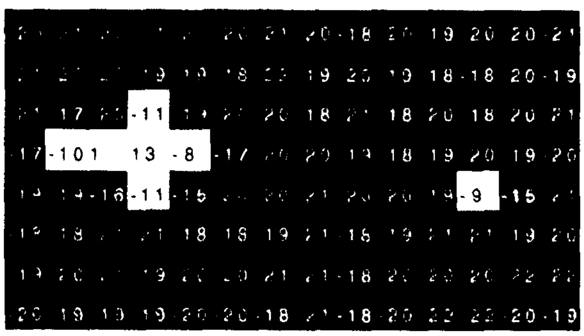

Figure 1: Point target response from C-band iii i AIRSAR image, showing position of 1 st azimuth ambiguity on the' right-hand side

POLARIMETRIC SAR MEASUREMENT'S

[n this section, the effects of noise on several common measures used in analysis of polarimetric SAK data are examined. It is assumed that the scattering matrix measurements have been 'symmetrized', i.e. that the HV and VH measurements have been averaged together, as is the case' for NASA/JPL AIRSAR data. Note that, after symmetrization, the noise power in [he HV measurement should be:

$$\sigma^n_{HV} = \frac{(\sigma^n_{HV} + \sigma^n_{VH})}{4}$$

COVARIANCE MATRIX

Forming cross-products between the elements of the measured matrix, M, yields the elements of the covariance matrix associated with the measurements. Under the assumption that the backscatter is reciprocal, the expected value of these six cross-products can be shown to be:

$$\left\langle M_{HH}M^*_{HH}\right\rangle = |S_{HH}|^2 + \sigma^n_{HH}$$
$$\left\langle M_{HV}M^*_{HV}\right\rangle = |S_{HV}|^2 + \sigma^n_{HV}$$
$$\left\langle M_{VV}M^*_{VV}\right\rangle = |S_{VV}|^2 + \sigma^n_{VV}$$
$$\left\langle M_{HH}M^*_{VV}\right\rangle = \left\langle S_{HH}S^*_{VV}\right\rangle$$
$$\left\langle M_{HH}M^*_{HV}\right\rangle = \left\langle S_{HH}S^*_{HV}\right\rangle$$
$$\left\langle M_{HV}M^*_{VV}\right\rangle = \left\langle S_{HV}S^*_{VV}\right\rangle$$

where the σ's are the noise powers in the HH, HV and VV measurements. The average values of these noise powers, if known, can be subtracted off cross-product measurements which have been averaged over areas. This corrects for the first-order noise effects. Spatial averaging also reduces the variance of higher order noise fluctuations.

STOKES MATRIX

Another way of representing the cross-products derived from the scattering matrix elements is in the Stokes matrix format, For reciprocal scatterers the Stokes matrix F is a 4x4 symmetric matrix, with the following elements:

$$F_{23} = 0.5\mathrm{Re}(M^*_{HH}M_{HV}) - 0.5\mathrm{Re}(M^*_{HV}M_{VV})$$

$$F_{24} = 0.5\mathrm{Im}(M^*_{HH}M_{HV}) - 0.5\mathrm{Im}(M^*_{HV}M_{VV})$$

$$F_{33} = 0.5(M_{HV}M^*_{HV}) + 0.5\mathrm{Re}(M^*_{HH}M_{VV})$$

$$F_{34} = 0.5\mathrm{Im}(M^*_{HH}M_{VV})$$

$$F_{44} = 0.5(M_{HV}M^*_{HV}) - 0.5\mathrm{Re}(M^*_{HH}M_{VV})$$

$$F_{11} = 0.25(M_{HH}M^*_{HH} + 2M_{HV}M^*_{HV} + M_{VV}M^*_{VV})$$

$$F_{12} = 0.25(M_{HH}M^*_{HH} - M_{VV}M^*_{VV})$$

$$F_{13} = 0.5\mathrm{Re}(M^*_{HH}M_{HV}) + 0.5\mathrm{Re}(M^*_{HV}M_{VV})$$

$$F_{14} = 0.5\mathrm{Im}(M^*_{HH}M_{HV}) + 0.5\mathrm{Im}(M^*_{HV}M_{VV})$$

$$F_{22} = 0.25(M_{HH}M^*_{HH} - 2M_{HV}M^*_{HV} + M_{VV}M^*_{VV})$$

When there is no signal present, i.e. the scattering matrix elements are all zero, the expected values of the Stokes matrix elements are:

$$\langle F_{11}\rangle = 0.25\left(\sigma^n_{HH} + 2\sigma^n_{HV} + \sigma^n_{VV}\right)$$

$$\langle F_{12}\rangle = 0.25\left(\sigma^n_{HH} - \sigma^n_{VV}\right)$$

$$\langle F_{13}\rangle = \langle F_{14}\rangle = \langle F_{23}\rangle = \langle F_{24}\rangle = \langle F_{34}\rangle = 0$$

$$\langle F_{22}\rangle = 0.25\left(\sigma^n_{HH} - 2\sigma^n_{HV} + \sigma^n_{VV}\right)$$

$$\langle F_{33}\rangle = \langle F_{44}\rangle = 0.5\sigma^n_{HV}$$

These are the first-order, additive noise powers to be corrected in the presence of a signal. Note that if the noise powers in each of the channels are equal (before symmetrization), only the F_{11}, F_{22}, F_{33} and F_{44} terms contain significant first-order noise terms. As for the covariance matrix, spatial averaging will reduce the variance of higher order noise fluctuations in the Stokes matrix measurements.

CORRELATION COEFFICIENTS

When forming a correlation coefficient between the HH and VV scattering matrix measurements, the following is calculated by averaging over an area:

$$\frac{\langle M_{HH} M^*_{VV}\rangle}{\sqrt{\langle M_{HH} M^*_{HH}\rangle\langle M_{VV} M^*_{VV}\rangle}}$$

This can be corrected for first-order noise effects by initially correcting the covariance matrix elements used to calculate the correlation coefficient. If this is not done, the estimated correlation coefficient will be lower than the actual one. The same applies to correlation coefficients formed from other combinations of scattering matrix elements. Again, spatial averaging will reduce the variance of higher order noise fluctuations in the correlation coefficient measurements.

POLARIZATION SIGNATURES

For any given radar receive and transmit polarization, the radar cross-section (RCS) can be calculated from the scattering matrix, S, via

$$\sigma_{pq} = 4\pi \left| \bar{q}^r\, S\, \bar{p}^t \right|^2$$

where q^r, p^t are polarization field vectors for the radar receive and transmit polarizations, respectively. This procedure is called polarization synthesis. Polarization signature plots are a useful tool for visualizing polarimetric scattering properties of a target. They represent the synthesized response of the target to all possible like-polarized or cross-polarized radar transmit/receive combinations. The polarization signature plots are given as functions of orientation and ellipticity angle, and are normalized with respect to the total power, F_{11}.

The scattering matrix model for Bragg scattering from an idealized rough surface, such as wind-blown water, is:

$$S = \begin{pmatrix} a & 0 \\ 0 & b \end{pmatrix} \quad \text{with } a,b \text{ real, } b > a > 0 \text{ and } \langle ab^*\rangle = ab$$

i.e., a scattering matrix with zero cross-polarized return, HH and VV returns which are completely correlated and zero phase difference between the HH and VV returns. A polarization signature corresponding to a typical Bragg scatterer is shown in Figure 2.

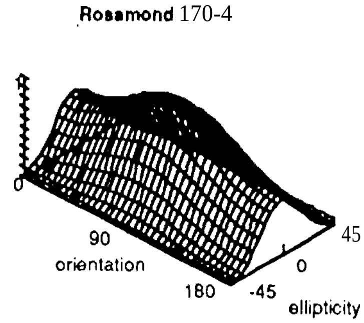

Figure 2: like-polarized polarization signature of a typical Bragg scatterer with high SNR

The polarization signature in Figure 2 shows the case where there is a high SNR (>18dB). Figure 3 contains an example of a polarization signature of a Bragg scatterer when the SNR is low (<7dB). Note that the pedestal (minimum level of the polarization signature) is significantly increased.

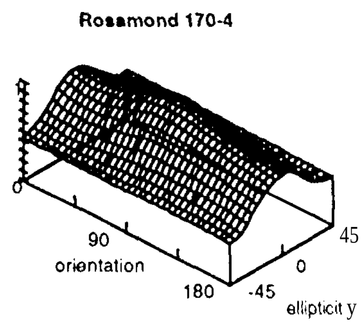

Figure 3: like-polarized polarization signature of a typical Bragg scatterer with low SNR

PHASE DIFFERENCE PLOTS

In Figure 4, HH-VV phase difference plots for the same scatterers examined in Figure 3 are shown. The presence of noise in the low SNR case broadens the distribution of the phase histogram but does not significantly alter the mode of the distribution.

SUMMARY AND DISCUSSION

The importance of knowledge of the noise power present in the HH, HV, VH and VV measurements made by a polarimetric SAR system has been demonstrated above. If the noise powers are known, they can be used to apply a first order correction to averaged covariance matrix or Stokes matrix values. If such first order noise corrections are not applied, several measures commonly derived from polarimetric SAR data may give erroneous results. Spatial averaging reduces the higher order fluctuations in the noise.

The noise power values as a function of range position in the image should, ideally, be provided with the data. In the absence of information on the noise powers, the user can get a crude estimate of an upper bound to the noise floor by examining the HV backscatter corresponding to a target demonstrating known Bragg scattering behavior (e.g. a water surface or a dry lake bed). This should give an estimate for the noise power in the HV measurements. For "symmetrized" data, if the noise powers were all equal before symmetrization, the HH and W noise powers would then be 3dB higher than the HV.

Also, it was shown that the first and brightest azimuth ambiguities (i.e. those closest to the main response) occur at predictable locations and with an HH-VV phase difference 180 degrees away from the phase difference associated with the main response. Thus, in polarimetric SAR data, very bright single-bounce or doubie-bounce scatterers will give rise to ambiguities in azimuth that look like fairly bright double-bcruncc or single-bounce scatterers, respectively.

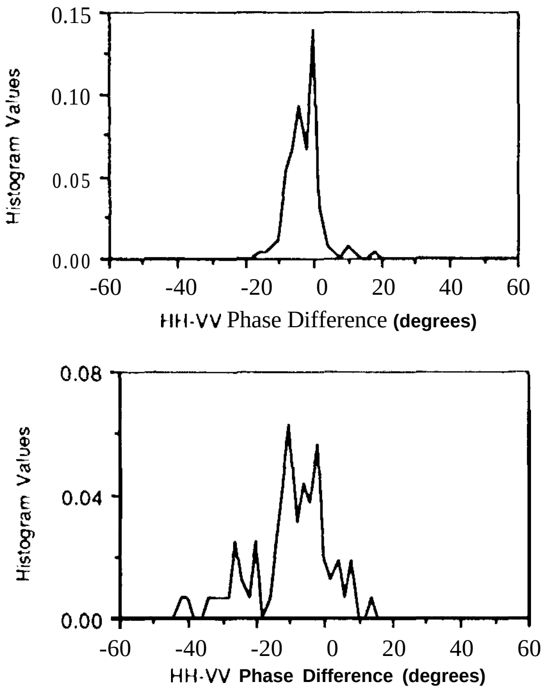

Figure 4: HH -VV phase difference histogram plots for typical Bragg scatterers with a) high SNR and b) low SNR

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REFERENCES

[1] Dubois, P.C., Evans, D. and van Zyl, J. J., "Approach to Derivation of Science Requirements for Calibration, IEEE Trans. on Geoscience and Remote Sensing, vol. 30, No. 6, Nov. 1992, pp. 1145-1149.